A Supporting Information: Mathematical Appendix

Note: The original manuscript contained two minor mistakes. On page 9, the original manuscript read "Secondly, the governments observe the rewards offered by the two lobbies and simultaneously choose to adjust or defect." It should have stated that "Secondly, the governments observe the rewards offered by their own lobbies and simultaneously choose to adjust or defect." This indicates that government i does not observe the reward offered by lobby j, and vice versa. Also, in Claim 5, V_j should have the same lower bound as in Claim 3: $V_j \in [\frac{c}{\delta} - b_j, c)$. We thank Michael Miller for catching these.

For the sake of brevity, in each of the proofs below, we prove the argument for a generic lobby, lobby i. A duplicate argument that considers decisions from the standpoint of the other lobby, lobby j, would mirror these results. The subscript i is removed to reduce clutter when there is no danger of confusion.

Proof of Claim 1, No Lobbies: Cooperation. In subgames in which one of the countries has previously defected, defecting is a best response for both countries. In period 1 and in any subgame in which neither government has previously defected, a government chooses to adjust if and only if:

$$EU(Adjust) \geq EU(Defect)$$
$$\frac{b-c}{1-\delta} \geq b$$
$$\delta b \geq c$$

Therefore, the claim holds. □

Proof of Claim 2, Internationally Benefiting Lobbies: Cooperation. In subgames in which one of the countries has previously defected, defecting is a best response for both countries if the rewards offered are less than c. The payoff for adjustment is $-c$ and the payoff for defecting is 0. The lobby's strategy of offering zero rewards in these subgames is also a best response. If the lobby offered a positive reward, less than c, their government will reject it and choose to defect. If the lobby offers a positive reward greater than or equal to c, their government will accept, but the lobby gains no additional payoff since the foreign government is still choosing to defect, and only incurs the cost of paying the reward.

In subgames without a previous defection, or on equilibrium path, the smallest reward the lobby can offer to induce adjustment satisfies:

$$EU(Adjust) \geq EU(Defect)$$
$$\tfrac{b-c+r}{1-\delta} \geq b$$
$$r \geq c - \delta b$$

If the lobby offers $r^* = c - \delta b$, their government is indifferent between adjustment and defecting, and it adjusts. The lobby's payoff from offering $r < r^*$ is V_i. For the lobby to want to at least offer r^*, it must be the case that:

$$EU(r = r^*) \geq EU(r < r^*)$$
$$\tfrac{V_i - c + \delta b}{1-\delta} \geq V_i$$
$$V_i \geq \tfrac{c}{\delta} - b$$

Trivially, the lobbies can do no better by offering a reward higher than r^*. Such a reward would be accepted, induce the same behavior by the governments, yet be more costly to pay. □

Proof of Claim 3, Domestic Lobbies: Cooperation with Punishment. Consider subgames in which one government has previously defected. The strategy calls for both governments to defect and both lobbies to offer zero rewards. In order to choose to adjustment, government i would have to be offered a reward satisfying:

$$EU(Adjust) \geq EU(Defect)$$
$$r - c \geq 0$$
$$r \geq c$$

Lobby i would only want to offer this reward if $V_i \geq c$. If $V_i < c$, then lobby i will not offer this reward, and government i will defect on the punishment path. Since government j defects, government i's best response is to also defect if lobby i offers $r_i = 0$.

On equilibrium path, the rest of this proof is identical to that of Claim 2. When $V_i \geq c$, in any subgame in which a government has previously defected, lobby i can always do better by offering a reward $r' \geq c$ which gets their government to adjust and yields lobby i a higher payoff.

Note that, for the conditions established in Claim 3, "... if and only if $V_i \in [\frac{c}{\delta} - b_i, c)$," $c \geq \frac{c}{\delta} - b_i$ implies $\frac{\delta}{1-\delta} > \frac{c}{b}$. Combined with the negative of the condition from Claim 1, that $c > \delta b_i$, this implies $\frac{c}{b_i} > \delta > \frac{c}{c+b_i}$. This condition is sufficient for there to be a "gap" between $\frac{c}{\delta} - b_i$ and c as depicted in Figures 3 and 4. □

Proof of Claim 4, Domestic Lobbies: Harmony. First, observe that if lobby i offers $r_i = c$, then government i is at worst indifferent between adjusting and defecting, regardless of whether government j adjusts or defects. When government j adjusts, the utility to government i of accepting/adjusting is $b - c + c$, which equals the utility to government i of defecting, b. When government j defects, the utility to government i of accepting/adjusting is $c - c = 0$, which is their utility to defecting. Subgame perfection rules out the ability of government i to try and do better by rejecting any offer/defecting if r_i is lower than some upper bound, $\overline{r} > c$. Lobby i could offer $r' = c + \epsilon$ which yields government i a strictly higher payoff to accepting/adjusting.

What is the smallest reward that lobby i can offer government i to adjust? On equilibrium path, lobby j offers $r_j = c$ and government j adjusts. In this case, the smallest reward i can offer, satisfies:

$$EU_i(Adjust) \geq EU_i(Defect)$$
$$\frac{b-c+r}{1-\delta} \geq \frac{b}{1-\delta}$$
$$r \geq c$$

Note that lobby i can do no better by offering a higher reward, since government i accepts/adjusts on equilibrium path and higher rewards would be more expensive for lobby i. For lobby i to want to follow this strategy and offer $r_i = c$, it must be the case that:

$$EU(Offer\ \ r = c) \geq EU(Offer\ \ r < c)$$
$$\tfrac{V_i - c}{1-\delta} \geq 0$$
$$V_i \geq c$$

This yields the condition in Claim 4. This proof was written in terms of lobby i; identical arguments for lobby j complete the proof.

□

Proof of Claim 5, Cooperation with Mixed Dyad. This proof is essentially combination of the proofs from Claims 2 and 3. First, note that the punishment strategies are Nash for both the lobbies and governments in the region outlined in Claim 5. For government i, defecting is a best response since government j defects. Lobby i also does not want to pay the necessary reward to induce government i to adjust, $r_i = c$. Similarly, government j does not want to adjust since government i is defecting, unless lobby j offers $r_j = c$, which lobby j is not willing to do.

The arguments for on equilibrium path behavior made in Claim 2 yield the lower bound for the condition on V^i, and the argument in Claim 3 yield the lower bound for the condition on V_j. The upper bound for the condition on V^j comes lobby j's inability to commit to refraining from rewarding government j for unilateral adjustment. These arguments were made in the proof to Claim 3.

□

Proof of Proposition 3. We prove the proposition by construction. First, recall that we have assumed throughout $V_i > c - b_i$ and $V_j > c - b_j$. This condition ensures that if cooperation can be enforced, all players can obtain strictly positive payoffs for some vector of strategies. It has to be shown that such a vector can be enforced. Following convention, a public randomization device is assumed to exist.

We construct such a vector of strategies building on Fudenberg and Maskin (1986). We propose the following strategy:

- At time $t = 1$, lobby i contributes r_i^*; government i adjusts if and only if lobby i contributes r_i^* (and the same for government and lobby j). This play is repeated indefinitely unless one player deviates.
- If some player deviates, it is labeled "defector." If a lobby and a government defect, the government is labeled "defector." Upon defection, the game continues as follows:
  - A punishment stage of T periods begins: contributions are zero, $r_i^{**} = r_j^{**} = 0$, and both governments defect.
  - When the punishment stage ends, a new stage begins. In this stage, the defector plays as in the original cooperation stage. For the governments, each government defects with a small probability $\epsilon \to^+ 0$ depending on the public randomization device's message. Lobbies offer $r^* - \varepsilon$ throughout, where $\varepsilon \to^+ 0$. This play is repeated indefinitely unless one player deviates.
- If any player defects during the T-period punishment, the punishment stage begins anew, except with a newly identified defector (based on the rules outlined above).

To prove the claim, it suffices to show that this strategy vector constitutes an SPNE.

First, consider the original cooperation stage. No player can benefit from defection because the immediate defection payoff, denoted by $X_i > 0$, is smaller than the payoff loss over time regardless of the value of T. To see why, note that since $\delta \to 1$, we have:

$$\frac{1}{1-\delta} v_i^* > X_i + \frac{\delta^{T+2}}{1-\delta} \left(v_i^* - Z_i \right),$$

where $Z_i > 0$ is the defector's strictly positive payoff reduction during the modified cooperation stage. Since $\frac{\delta^{T+2}}{1-\delta} \to \infty$, any finite value X_i can be discounted. With $\delta^{T+2} \to 1$, the comparison simplifies into $v_i^* > v_i^* - Z_i$, which must hold.

Second, consider the punishment stage. The payoff from equilibrium play during the T periods is zero, so it is clear that no government can benefit from defection as long as the lobbies are not defecting. For a lobby, the payoff from equilibrium play is also zero. The only potentially profitable defection is to increase the reward r_i so much that government i adjusts. However, this requires that the government respond to the optimal defection, denoted by $\hat{r}_i$, by defecting and restarting the punishment stage. To see that this is not possible, first let the government's gain from defection be denoted by $Y_i(\hat{r}_i) > 0$. The government prefers not to defect whenever

$$\frac{\delta^K}{1-\delta}\left(v_i^* - \tilde{Z}_i\right) > Y_i(\hat{r}_i) + \frac{\delta^{T+2}}{1-\delta}\left(v_i^* - Z_i\right),$$

where $\tilde{Z}_i$ is the government's payoff loss from the modified cooperation stage relative to the original cooperation stage as a "non-defector." By construction, $\tilde{Z}_i < Z_i$. With $\delta \to 1$, the government obtains a strictly negative payoff change from defection. The proof is almost identical to that used to conduct the comparison in the cooperation stage. But then no lobby can profitably deviate either, because a lobby's deviation produces a zero payoff whenever the government does not respond.

The proof for the final stage is virtually identical to the proof for the original cooperation stage.

□